

Signals and Systems — Tutorial 11

Modulation

Tutorial information

- It is expected that you attempt the prework questions before coming to the tutorial each week.
- Quizzes may be attempted during the last half hour of the tutorial.

Prework questions

A: Amplitude Modulation

A signal $u(t)$ with bandwidth 200–5000 Hz is to be transmitted wirelessly using amplitude modulation. Write down the steps needed to modulate and demodulate the signal.

Tutorial questions

1: Frequency Division Multiplexing

The Medium Waveband covers frequencies from 500 to 1500 kHz. Calculate the number of simultaneous AM transmissions that can be accommodated in this band, if the bandwidth of each modulating signal is 4 kHz and a gap of 3 kHz is required between sidebands of adjacent signals to prevent crosstalk interference.

2: Carrier Phase Error

Consider an AM wave

$$y(t) = u(t) \cos(2\pi f_c t)$$

where $u(t)$ is the information signal. Recall that the receiver consists of a product modulator followed by a low-pass filter. The product modulator multiplies the received waveform by a locally generated sinusoid of frequency f_c . Suppose that the locally generated sinusoid has a phase difference of ϕ with the original carrier, as shown in Figure 1.

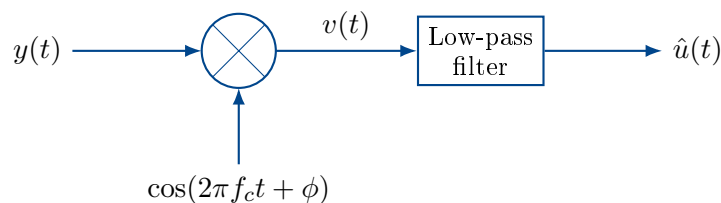


Figure 1: Receiver with a phase error ϕ in the local carrier.

Show that the demodulated signal is $\hat{u}(t) = u(t) \cos \phi$, when the gain of the low-pass filter is appropriately chosen. What happens when $\phi = 90^\circ$?

3: Standard AM

The AM signal from lectures, $y(t) = u(t) \cos(\omega_c t)$, is called *double sideband suppressed carrier* (DSB-SC) modulation. Standard AM, used for AM radio broadcasts, adds the carrier to the transmitted signal:

$$y(t) = (A_c + u(t)) \cos(\omega_c t),$$

where $A_c > 0$ is the carrier amplitude. The modulation index is $m = \max_t |u(t)|/A_c$.

1. Suppose $U(j\omega) = 0$ for $|\omega| > W$, with $W < \omega_c$. Find $Y(j\omega)$ in terms of U . Sketch $U(j\omega)$ and $Y(j\omega)$ for a $U(j\omega)$ of your choice. How does $Y(j\omega)$ differ from the spectrum of the DSB-SC signal?
2. An amplitude modulated signal has a waveform defined by

$$y(t) = [10 + a \cos(2\pi f_x t)] \cos(18 \times 10^6 \pi t).$$

What are the amplitude and frequency (in Hz) of the unmodulated carrier?

3. If $a = 3$ and $f_x = 1.5$ kHz calculate the modulation index and sketch the waveform. Repeat the sketch for $a = 15$.
4. The receiver passes $y(t)$ through a full-wave rectifier, with output $|y(t)|$, followed by a low-pass filter. Assume $A_c + u(t) \geq 0$ for all t . The Fourier series of $|\cos(\omega_c t)|$ has DC term $2/\pi$ and harmonics at $2\omega_c, 4\omega_c, \dots$. Show that a low-pass filter with a suitable cutoff recovers $\frac{2}{\pi}(A_c + u(t))$. What condition on m guarantees $A_c + u(t) \geq 0$? What is the output for the $a = 15$ signal of part 3?

5. Write the $a = 3$ signal of part 3 as a sum of three sinusoids. Find the fraction of the average power of $y(t)$ that is in the two sidebands. Repeat for $a = 10$.

4: Square-Law Modulator

A simple diode-resistor *square-law* circuit can be made with input u V and output y V such that $y = u^2$. The input to the square-law circuit is

$$u(t) = 2 + a_1(t) + a_2(t)$$

where $a_1(t) = b \cos(2\pi f_x t)$ is a sinusoidal modulating signal and $a_2(t) = \cos(2\pi f_c t)$ is a unit amplitude carrier signal. Show that if the output $y(t)$ is fed into a filter which passes only a suitable frequency band centered on f_c , then the filter output is a standard AM signal with carrier frequency f_c . What is the modulation index of the AM signal?