

Signals and Systems — Tutorial 8

Filtering

Tutorial information

- It is expected that you attempt the prework questions before coming to the tutorial each week and submit your *handwritten* solutions to your tutor within the first 15 minutes of your tutorial.
- Not all questions are equally difficult. Difficult questions are marked with a ★.
- Questions marked **Extension** go beyond the assessable content for this course and are provided to give a deeper theoretical background and understanding.
- You may not be able to complete all questions within the tutorial time. It is *strongly* recommended that you attempt all questions in your own time and ask your tutor if you are stuck.

Prework questions

A noisy signal, $f(t)$, is shown in Figure 1 and filtered using the filters $F_1(s)$, $F_2(s)$ and $F_3(s)$ whose Bode diagrams are shown in Figure 2 where F_1 is in red, F_2 is in green and F_3 is in orange. The output of each filter is shown in Figure 3 by $y_g(t)$, $y_b(t)$ and $y_r(t)$ given by the lime, blue and red lines respectively.

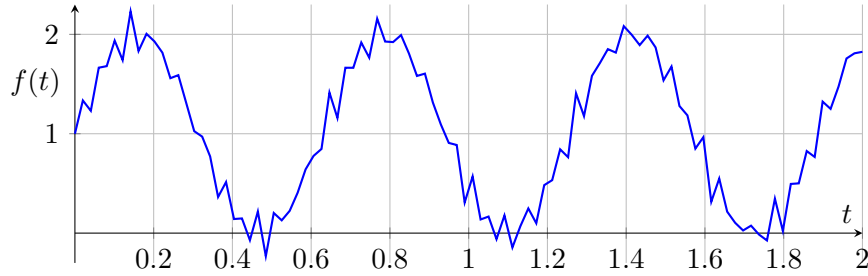


Figure 1: Measured signal, $f(t)$

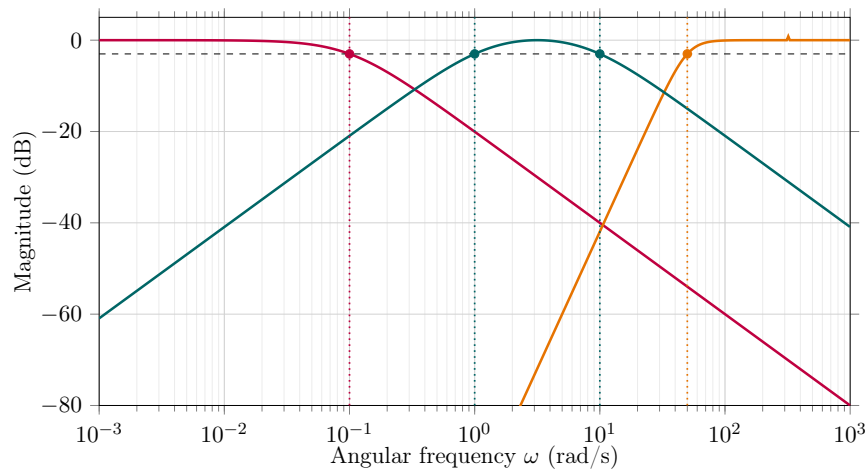


Figure 2: Bode plots of the filters $F_1(s)$, $F_2(s)$ and $F_3(s)$.

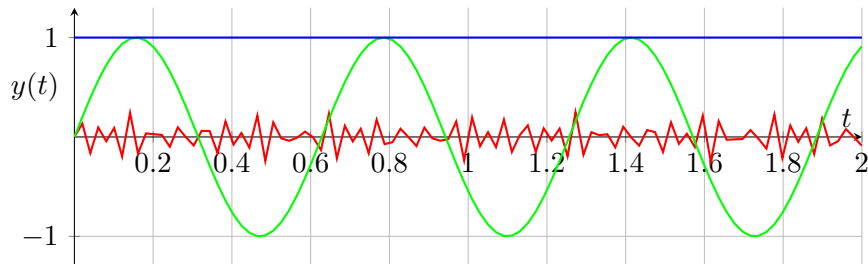


Figure 3: Filtered outputs $y_g(t)$ (lime), $y_b(t)$ (blue) and $y_r(t)$ (red) of the measured signal $f(t)$.

Identify whether F_1 , F_2 and F_3 are low-pass, high-pass, band-pass or band notch filters and match the filtered outputs $y_g(t)$, $y_b(t)$ and $y_r(t)$, shown in Figure 3 to which filter was used to filter the signal, $f(t)$.

Tutorial questions

1: Poles and zeros

1. For each of the filters F_1 , F_2 and F_3 in Figure 2 identify the location of the poles and zeros of the filter and hence find the transfer functions for each of the filters.
2. Find the time domain differential equation for the filters F_1 and F_2 and determine their impulse responses.
3. For each of the filters find the frequency ω_c such that a sinusoidal input $A \sin(\omega_c t)$ is attenuated by a factor of $1/\sqrt{2}$. Demonstrate this by using convolution for at least one of the filters.

2: Butterworth filters

Suppose the orange curve in Figure 2 is the frequency response of an order n Butterworth filter.

1. Determine the order n of the filter.
2. Find the damping coefficients ζ_k and give the transfer function of the filter.
3. Find the phase of the filter at the cut-off frequency, ω_c .

3: RLC Filter

Consider the RLC series circuit in Figure 4 with input $v(t)$. Determine if the circuit is a high-pass, low-pass or band-pass filter if the output is **(a)** the voltage across the inductor V_L , **(b)** the voltage across the capacitor V_C or **(c)** the current i in the circuit and determine the cut-off frequencies for each case.

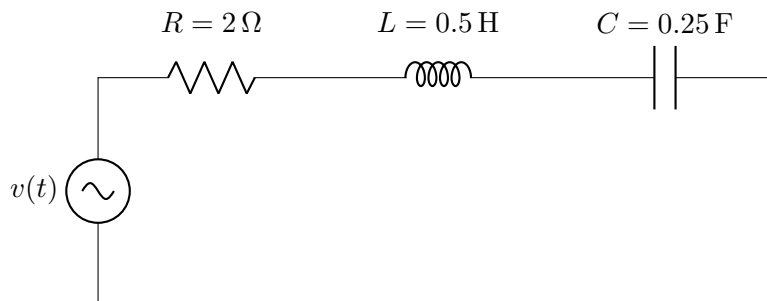


Figure 4: RLC series circuit.

4: Filters in Cascade

Consider the circuit (two first order filter stages connected in cascade) in Figure 5.

1. Using the transfer functions

$$G_A(s) = \frac{V_A(s)}{U_A(s)} \text{ and } G_B(s) = \frac{V_B(s)}{U_B(s)}$$

Show that stage A is a high-pass filter, stage B is a low-pass filter and determine their DC gains.

- Determine values of R , L and C so that the circuit has the frequency response as shown by the green curve in Figure 2.

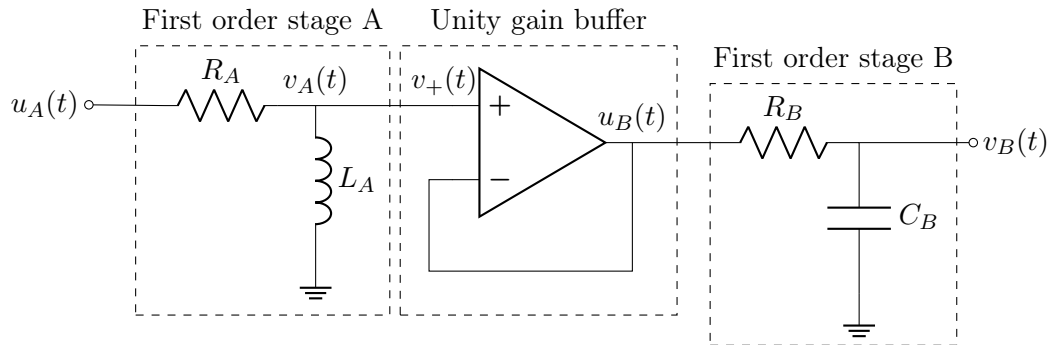


Figure 5: Two first order filter stages (implemented as RL and RC circuits) connected in cascade by a unity-gain voltage buffer i.e. a second order band-pass filter

5: Filter Design - Tunable Suspension

Mechanical engineers have designed a tunable suspension system for a car based on a mass-spring-damper where the magnitude of the damping coefficient is adjustable. The suspension system satisfies

$$m\ddot{x}(t) + b\dot{x}(t) + kx(t) = F(t) \quad (1)$$

where m is the mass of the car, b is the damping coefficient, k is the spring constant, $x(t)$ is the vertical displacement of the car and $F(t)$ is some external force.

- Show that (1) is a low-pass filter and determine its DC gain in terms of m , b and k .
- Assuming that (1) is a Butterworth filter find its cutoff frequency in terms of m , b and k .
- The car is travelling at 36 km/h over a very rough road with potholes every 10 cm. In order not to damage the car this external force needs to be attenuated by 100 dB. However, speed-bump regulations specify that the external force from speed-bumps placed at one metre intervals can be attenuated by no more than 25 dB when travelling at 10 m/s. Determine the minimum order of a Butterworth filter which meets these specifications and give its transfer function.
- To build this filter specify how many mass-spring-damper systems must be put in cascade with damping coefficients b_i and spring constants k_i and give values for each which realises the filter from above assuming the mass of the car is 1000 kg.
- The mechanical engineers want to use identical springs and dampers for each of the mass-spring-damper systems used. Is this possible? Determine if the mechanical engineers should have paid more attention in ELEC2302.

6: Smoothing Filter

Recall from Tutorial 5 that a DC power supply takes a 50 Hz mains sinusoid, passes it through the full-wave rectifier of Tutorial 3 and then through the LC filter of Figure 6 to remove the ripple where $R \approx 0.5 \Omega$ is the winding resistance of the inductor.

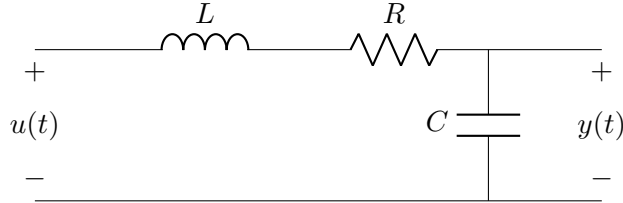


Figure 6: Smoothing filter. The input $u(t)$ is the rectifier output and $y(t)$ is the supply output.

The rectifier output is $u(t) = A |\sin(\omega_m t)|$ with $\omega_m = 2\pi \times 50$ rad/s. A measure of the ripple is the RMS of the non-constant parts of the signal i.e.

$$RMS(f_{ripple}) = \sqrt{\frac{1}{T} \int_0^T |f(t) - c_0|^2 dt}$$

Recall the Fourier series of a rectified sine wave is

$$A |\sin(\omega_0 t)| = \sum_k c_k e^{j2\omega_0 kt} = \sum_k \frac{2A e^{j2\omega_0 kt}}{\pi (1 - 4k^2)}$$

1. Find the magnitude of the ripple of the unfiltered rectified wave and the ratio of the ripple to the DC component of the rectified wave.
2. Find the transfer function $H(j\omega)$ from $u(t)$ to $y(t)$ for the circuit in Figure 6. Show that the filter is a second-order Butterworth filter if and only if $R = \sqrt{2L/C}$ with cut-off frequency $1/\sqrt{LC}$.
3. Recall that for a periodic signal the Fourier transform of a periodic signal is a train of impulses,

$$U(j\omega) = 2\pi \sum_k c_k \delta(\omega - k\omega_0),$$

Find the cut-off frequency required for a second order Butterworth filter so that the ratio of the ripple to the DC-component of the filtered signal is at most 10%.

4. Give values for L and C that achieve this cutoff frequency and so that the circuit in Figure 6 is a Butterworth filter.