Signals and Systems — Tutorial 5

Transfer Functions and Frequency Response

Tutorial information

- It is expected that you attempt the prework questions before coming to the tutorial each week and submit your *handwritten* solutions to your tutor within the first 15 minutes of your tutorial.
- Not all questions are equally difficult. Difficult questions are marked with a $\star$.
- Questions marked **Extension** go beyond the assessable content for this course and are provided to give a deeper theoretical background and understanding.
- You may not be able to complete all questions within the tutorial time. It is *strongly* recommended that you attempt all questions in your own time and ask your tutor if you are stuck.

Prework questions

A

The series RLC circuit from lectures has transfer function from source voltage $u(t)$ to capacitor voltage $y(t)$ given by

$$G(j\omega) = \frac{25}{(j\omega)^2 + 6j\omega + 25}.$$

Evaluate $G(j\omega)$ at $\omega = 0.5$, $\omega = 5$ and $\omega = 50$ rad/s. For each, give the magnitude in decibels and the phase in degrees.

B

1. Express as a level in decibels: an amplitude ratio of 2, an amplitude ratio of $1/\sqrt{2}$, an amplitude ratio of 0.015, and a power ratio of 2.
2. Express as a ratio: an amplitude gain of -3 dB, an amplitude gain of -40 dB, and an amplitude gain of $+6$ dB.
3. Two amplifiers with amplitude gains of 12 dB and -4 dB are connected in cascade. Give the overall gain in decibels and as a ratio.

Tutorial questions

1: Smoothing Filter for a DC Power Supply

A DC power supply takes a 50 Hz mains sinusoid, passes it through the full-wave rectifier of Tutorial 3, then through the LC filter of Figure 1 to remove the ripple. $R \approx 0.5\,\Omega$ models the winding resistance of the inductor.

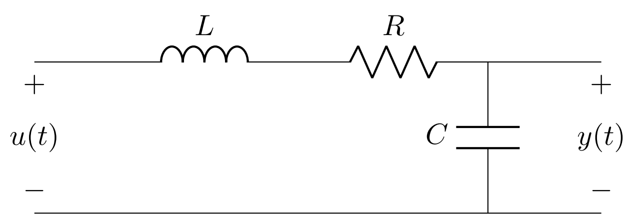

Figure 1: Smoothing filter. The input $u(t)$ is the rectifier output and $y(t)$ is the supply output.

1. The rectifier output is $u(t) = A\,|\sin(\omega_m t)|$ with $\omega_m = 2\pi \times 50$ rad/s. Using the Fourier series of the rectified sine from Tutorial 3, write down the DC term and the amplitude of the lowest-frequency ripple component, and state the frequency of that component. Show that the ratio of ripple amplitude to DC level at the filter input is 2/3, independent of A.

2. The Fourier transform of a periodic signal is a train of impulses,

$$U(j\omega) = 2\pi \sum_k c_k\, \delta\left(\omega - k\omega_0\right),$$

   where ω_0 is the fundamental and c_k are the complex Fourier coefficients. Write down the c_k of $u(t)$ when $A = 1$ and hence the weight of each impulse. Plot the magnitude and phase of $U(j\omega)$ for $-6\omega_m \le \omega \le 6\omega_m$. The ratio of the first ripple line to the DC line is not the 2/3 of Part 1. Explain.

3. Find $G(j\omega) = Y(j\omega)/U(j\omega)$. Put $G(j\omega)$ in the standard second-order form and identify ω_n, ζ and $G(0)$.

4. Sketch $|G(j\omega)|$ in decibels against $\log_{10}\omega$. Mark on the same axes the frequency of the ripple component found in Part 1.

5. The supply must deliver a DC level with ripple no greater than 1% of that level. Show that this requires $|G(j2\omega_m)| \le 0.015$. Using the Bode diagram from part 4, find the largest ω_n that meets the specification. If $L = 10$ mH, choose a value of C that achieves the specification.

6. With the L and C from in part 5, express ζ in terms of R and find the value of R below which $|G(j\omega)|$ has a peak. Take $R = 0.5\,\Omega$ and find the ζ. Determine whether $|G(j\omega)|$ exceeds $|G(0)|$ at any frequency, and if it does, give the frequency.

   The supply is now driven by an engine-driven alternator instead of the mains. On start-up the alternator runs up from rest, so the ripple frequency $2\omega_m$ sweeps upward from zero to its final value. Describe what happens to the output ripple during the run-up. What would have to change to remove the effect?

7. Using $Y(j\omega) = G(j\omega)U(j\omega)$ and the sifting property, write down the weight of each line of $Y(j\omega)$. Evaluate the DC line and the first three ripple lines and plot the magnitude of $Y(j\omega)$ on the axes of part 2.

8. The single filter is replaced by two identical sections in cascade, buffered so that the second section does not load the first. Write the overall transfer function. Show that the corner frequency of each section may now be placed a factor of $0.015^{-1/4}$ higher than the single-section value found in part 5. Comment on what happens to the peak of part 6 when the two sections are cascaded.

Use the remaining time to catch up on tutorial 3 & 4 if you still haven't completed them, in preparation for quiz 2!