

ELEC2302/9302 Signals and Systems – Quiz 3 practice questions, weeks 8 to 11¹

How to use this question bank: *don't* work through the questions from beginning to end. Start by doing one question from each group, identify the areas where you are less confident, and work on those first.

1. The Laplace transform (week 8)**Laplace transform: definition, ROC and table pairs**

1. What is the region of convergence of the Laplace transform of $u(t) = (3e^{-4t} - 2e^t)H(t)$?
 - (a) $\text{Re}(s) > 1$
 - (b) $-4 < \text{Re}(s) < 1$
 - (c) $\text{Re}(s) > 0$
 - (d) $\text{Re}(s) > -4$
2. The signal $u(t) = e^{(-2+5j)t}H(t)$ has Laplace transform $U(s) = \frac{1}{s+2-5j}$. What is its region of convergence?
 - (a) $\text{Re}(s) > 5$
 - (b) $\text{Re}(s) > -2$
 - (c) $|s| > \sqrt{29}$
 - (d) $\text{Re}(s) > 2$
3. The centered pulse $p_2(t)$ is delayed by 3 s, giving $u(t) = p_2(t-3)$. What is $U(s)$?
 - (a) $\frac{e^{-s} - e^{-5s}}{s}$
 - (b) $\frac{e^{-3s}(1 - e^{-2s})}{s}$
 - (c) $\frac{e^{-4s} - e^{-2s}}{s}$
 - (d) $\frac{e^{-2s} - e^{-4s}}{s}$
4. What is the Laplace transform of $u(t) = \cos(3t)\cos(t)H(t)$?
 - (a) $\frac{1}{2} \left(\frac{s}{s^2+9} + \frac{s}{s^2+1} \right)$
 - (b) $\frac{s^2}{(s^2+9)(s^2+1)}$
 - (c) $\frac{1}{2} \left(\frac{s}{s^2+16} + \frac{s}{s^2+4} \right)$

¹Some questions are adapted from B. P. Lathi and R. A. Green, *Linear Systems and Signals*, 3rd ed. (chapters 4 and 7), from ELEC3004 *Signals, Systems and Control* at the University of Queensland (2017), and from S. Boyd, EE102 *Introduction to Signals and Systems*, Stanford University.

(d) $\frac{1}{2} \left(\frac{s}{s^2 + 16} - \frac{s}{s^2 + 4} \right)$

5. A system has transfer function $G(s) = 5$ (a constant). What is its impulse response $h(t)$?

(a) $5H(t)$

(b) It has no impulse response because it is not a dynamic system.

(c) 5 for all t

(d) $5\delta(t)$

6. Write $\cos(4t)H(t) = \frac{1}{2}e^{4jt}H(t) + \frac{1}{2}e^{-4jt}H(t)$ and use linearity with $\mathcal{L}\{e^{s_0 t}H(t)\} = 1/(s - s_0)$. The result is

(a) $\frac{4}{s^2 + 16}$

(b) $\frac{1}{s^2 + 16}$

(c) $\frac{s}{s^2 + 16}$

(d) $\frac{s}{s^2 - 16}$

Laplace transform properties

7. Which signal has Laplace transform $\frac{e^{-3s}}{s + 1}$?

(a) $e^{-(t+3)}H(t)$

(b) $e^{-(t-3)}H(t - 3)$

(c) $e^{-t}H(t - 3)$

(d) $e^{-(t-3)}H(t)$

8. An LTI system, initially at rest, has step response $(e^{-t} - e^{-2t})H(t)$. What is its impulse response $h(t)$?

(a) $\left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right)H(t)$

(b) $(2e^{-2t} - e^{-t})H(t)$

(c) $(e^{-t} - e^{-2t})H(t)$

(d) $(e^{-t} - 2e^{-2t})H(t)$

9. Let $y(t) = (e^{-t}H(t)) * (e^{-3t}H(t))$. Use the Laplace transform to find $y(1)$.

Answer: _____

10. What is the Laplace transform of $u(t) = e^{2t} \sin(3t) H(t)$?

(a) $\frac{3}{s^2 - 4s + 7}$

(b) $\frac{3}{s^2 - 4s + 13}$

(c) $\frac{3}{s^2 + 4s + 13}$

(d) $\frac{s - 2}{s^2 - 4s + 13}$

11. What is the Laplace transform of $y(t) = \int_0^t \cos(3\tau) d\tau$ for $t \geq 0$?

(a) $\frac{1}{s^2 + 9}$

(b) $\frac{3}{s^2 + 9}$

(c) $\frac{1}{s(s^2 + 9)}$

(d) $\frac{s^2}{s^2 + 9}$

12. A signal $q(t)$, zero for $t < 0$, has $Q(s) = \frac{2 - 4e^{-2s}}{s + 3}$. Find $q(3)$.

Answer: _____

13. A system has transfer function $G(s) = k \left(1 + \frac{1}{\tau_i s} \right)$ with $k = 2$ and $\tau_i = 0.5$ s. Its input is $u(t) = H(t)$. Using the integral property, find the output $y(t)$ at $t = 3$ s.

Answer: _____

Solving differential equations with initial conditions

14. $y(t)$ satisfies $\ddot{y} + 16y = 0$ for $t \geq 0$, with $y(0) = 0$ and $\dot{y}(0) = 2$. What is $Y(s)$?

(a) $\frac{2}{s(s^2 + 16)}$

(b) $\frac{-2}{s^2 + 16}$

(c) $\frac{2}{s^2 + 16}$

(d) $\frac{2s}{s^2 + 16}$

15. Solve $\ddot{y} + 5\dot{y} + 6y = 6u(t)$ with $u(t) = H(t)$, $y(0^-) = 2$, $\dot{y}(0^-) = -1$. Give $y(1)$.

Answer: _____

16. Solve $\ddot{y} + 4\dot{y} + 4y = 2u(t)$ with $u(t) = H(t)$, $y(0^-) = 0.5$, $\dot{y}(0^-) = 0$. For $t \geq 0$, $y(t) =$

(a) $0.5 + 0.5e^{-2t} + te^{-2t}$

(b) $0.5 - 0.5e^{-2t} - te^{-2t}$

(c) 0.5

(d) $0.5e^{-2t} + te^{-2t}$

17. A current source $u(t) = tH(t)$ A drives a capacitor $C = 0.25$ F in parallel with an inductor $L = 1$ H. Both start with zero energy. Find the maximum value of the voltage $y(t)$ across them for $t \geq 0$, in volts.

Answer: _____

Initial and final value theorems

18. A system has $G(s) = \frac{120}{s^2 + 6s}$. Use the final value theorem to find $\lim_{t \rightarrow \infty} h(t)$ for the impulse response $h(t)$.

Answer: _____

19. The system $G(s) = \frac{s+1}{s+10}$ is driven by a unit step $u(t) = H(t)$ from rest. What are $y(0^+)$ and $\lim_{t \rightarrow \infty} y(t)$?

(a) $y(0^+) = 0, y(\infty) = 0.1$

(b) $y(0^+) = 0.1, y(\infty) = 1$

(c) $y(0^+) = 1, y(\infty) = 0$

(d) $y(0^+) = 1, y(\infty) = 0.1$

20. An RC lowpass with $G(s) = \frac{1}{2s+1}$ (time in seconds) is driven by the ramp $u(t) = tH(t)$ V, starting from rest. Find $\lim_{t \rightarrow \infty} (u(t) - y(t))$ in volts.

Answer: _____

21. For which one of these transfer functions does the final value theorem correctly give $\lim_{t \rightarrow \infty} h(t)$ for the impulse response?

(a) $\frac{12}{s-4}$

(b) $\frac{12}{s^2+4}$

(c) $\frac{12}{s^4+4}$

(d) $\frac{12}{s(s+4)}$

2. System analysis with the Laplace transform (week 9)

Transfer functions, poles and stability

22. Classify $G(s) = \frac{12}{s^2+4}$.

(a) BIBO stable but not exponentially stable

(b) Marginally stable and not BIBO stable

(c) Unstable

(d) Exponentially stable

23. A transfer function $G(s)$ has poles at 0 and $-0.2 \pm j$, with no pole-zero cancellation. Which description is correct?
- (a) Marginally stable, but not BIBO stable
 - (b) Marginally stable and BIBO stable
 - (c) Exponentially stable
 - (d) Unstable
24. Which transfer function is marginally stable?
- (a) $\frac{s+2}{(s^2+4)(s+1)}$
 - (b) $\frac{s-2}{(s+1)(s+3)}$
 - (c) $\frac{s+2}{s^2(s+3)}$
 - (d) $\frac{s+2}{(s^2+4)^2}$
25. Classify $G(s) = \frac{12}{s^4+4}$. (Hint: $s^4+4 = (s^2+2s+2)(s^2-2s+2)$.)
- (a) Exponentially stable, since all denominator coefficients are positive
 - (b) BIBO stable but not exponentially stable
 - (c) Marginally stable, with poles on the imaginary axis like s^2+4
 - (d) Unstable
26. For $G(s) = \frac{1}{s^2+9}$ with zero initial conditions, which bounded input produces an unbounded output?
- (a) $\cos(3t)H(t)$
 - (b) $H(t)$
 - (c) $\cos(9t)H(t)$
 - (d) $e^{-3t}H(t)$

Partial fractions and inverse transforms

27. Find the impulse response of $G(s) = \frac{s+2}{s(s+1)}$ at $t = 1$ s.

Answer: _____

28. Write $Y(s) = \frac{6}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$. Find A .

Answer: _____

29. What is the inverse Laplace transform of $Y(s) = \frac{2s+5}{s^2+2s+1}$?

- (a) $2\delta(t) + 3te^{-t}H(t)$
- (b) $(2 + 3t)e^{-t}H(t)$
- (c) $(3 + 2t)e^{-t}H(t)$
- (d) $(2 + 5t)e^{-t}H(t)$

30. The inverse transform of $Y(s) = \frac{3s + 1}{(s + 1)(s^2 + 2s + 5)}$ has the form

$$y(t) = Ae^{-t} + e^{-t}(B \cos 2t + C \sin 2t), \quad t \geq 0.$$

Find C .

Answer: _____

31. $G(s) = \frac{s + 3}{(s + 1)(s + 3.1)}$ has a zero very close to a pole. Find the coefficient of $e^{-3.1t}$ in the impulse response $g(t)$.

Answer: _____

32. Find $y(3)$ where $Y(s) = \frac{4e^{-s}}{(s + 1)(s + 3)}$.

Answer: _____

33. $G(s) = \frac{s + 4}{s^2 + 3s + 2}$ is driven by $u(t) = e^{-2t}H(t)$ with zero initial conditions. Find $y(1)$.

Answer: _____

34. An RC lowpass with $G(s) = \frac{1}{s + 1}$ starts from rest and is driven by $u(t) = (1 - e^{-3t})H(t)$ V. Find $y(1)$ in volts.

Answer: _____

35. Two identical RC sections, each with transfer function $\frac{2}{s + 2}$, are connected through an ideal unity-gain buffer, so the overall transfer function is $\frac{4}{(s + 2)^2}$. A unit step is applied from rest. Find the output at $t = 1$ s.

Answer: _____

36. A series RLC circuit ($L = 1$ H, $C = 0.25$ F, $R = 2$ Ω , output across C , starting from rest) has $G(s) = \frac{4}{s^2 + 2s + 4}$. Which input $u(t)$ makes the output exactly $y(t) = (1 - e^{-4t})H(t)$?

- (a) $u(t) = \delta(t) + (1 - 3e^{-4t})H(t)$
- (b) $u(t) = (1 - 3e^{-4t})H(t)$
- (c) No such input exists, because the circuit is underdamped
- (d) $u(t) = (1 - e^{-4t})H(t)$

Zeros

37. $G(s)$ has poles at -2 and -5 , a single zero at -1 , and DC gain 3. The input is $u(t) = e^{-t}H(t)$, starting from rest. What is $y(t)$?
- (a) $(e^{-2t} - e^{-5t})H(t)$
 - (b) $y(t) = 0$, because the zero at -1 blocks e^{-t}
 - (c) $10(e^{-2t} - e^{-5t})H(t)$
 - (d) $y(t)$ contains a nonzero e^{-t} term as well as e^{-2t} and e^{-5t} terms
38. $G(s) = 1 + \frac{1}{s} + 0.25s$. Which describes the straight-line Bode magnitude?
- (a) 0 dB/decade below 2 rad/s, then +40 dB/decade
 - (b) -20 dB/decade below 2 rad/s, then +20 dB/decade
 - (c) -20 dB/decade below 2 rad/s, then +40 dB/decade
 - (d) -20 dB/decade below 2 rad/s, then 0 dB/decade
39. A filter $G(s) = K \frac{s+z}{s+5}$ has DC gain 1 and must have a high-frequency gain of -30 dB. Where must the zero be? Give z in rad/s (the zero is at $s = -z$).

Answer: _____

40. A 50 Hz hum notch filter is $G(s) = \frac{s^2 + \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ with $\omega_n = 2\pi \cdot 50$ rad/s and $\zeta = 0.5$. Find $|G(j\omega)|$ at 25 Hz.

Answer: _____

3. Filter design (week 10)**Filter specifications and first- and second-order filters**

41. Why can an ideal lowpass filter $G(j\omega) = p_{2\omega_c}(\omega)$ not be built as a real-time circuit?
- (a) Its gain is infinite at ω_c
 - (b) Its phase is not linear
 - (c) Its impulse response $\sin(\omega_c t)/(\pi t)$ is nonzero for $t < 0$, so it is noncausal
 - (d) It has a pole in the right half plane
42. At the -3 dB frequency of a first-order RC lowpass, compared with DC, the output
- (a) amplitude is $1/2$ and power is $1/4$
 - (b) amplitude is $1/\sqrt{2}$ and power is $1/2$
 - (c) amplitude and power are both 0.7
 - (d) amplitude is $1/2$ and power is $1/\sqrt{2}$
43. Which transfer function is a bandpass filter?

(a) $\frac{10s}{s^2 + 10s + 100}$

(b) $\frac{100}{s^2 + 10s + 100}$

(c) $\frac{s^2 + 100}{s^2 + 10s + 100}$

(d) $\frac{s^2}{s^2 + 10s + 100}$

44. Find the -3 dB frequency of the second-order lowpass $G(s) = \frac{400}{s^2 + 20s + 400}$, in rad/s.

Answer: _____

45. A first-order highpass $G(s) = \frac{s}{s + \omega_c}$ with $\omega_c = 2\pi \cdot 100$ rad/s is used to suppress a 50 Hz hum. By how many dB is the 50 Hz hum attenuated? (Give a positive number.)

Answer: _____

46. A first-order lowpass $G(s) = \frac{k}{s + a}$ with $k, a > 0$ must have DC gain $+12$ dB and a gain 3 dB below the DC gain at $f = 1$ kHz. Find k (units rad/s).

Answer: _____

47. Which transfer function is an exponentially stable allpass filter ($|G(j\omega)| = 1$ for all ω)?

(a) $\frac{s}{s + 3}$

(b) $\frac{3}{s + 3}$

(c) $\frac{s + 3}{s - 3}$

(d) $\frac{s - 3}{s + 3}$

48. The allpass filter $G(s) = \frac{10 - s}{s + 10}$ is driven by the input $u(t) = \cos(10t)$, applied for all time. The output is

(a) $-\cos(10t)$

(b) $\cos(10t)$

(c) $\sin(10t)$

(d) $-\sin(10t)$

49. The system $y(t) = u(t - 2)$ has transfer function $G(s) = e^{-2s}$. Which describes its frequency response?

(a) It has a pole at $s = -2$, so $|G|$ falls at -20 dB/decade above 2 rad/s

(b) $|G(j\omega)| = 1$ and $\angle G(j\omega) = -2\omega$ rad: an allpass

- (c) $|G(j\omega)| = e^{-2\omega}$: a lowpass
- (d) $|G(j\omega)| = 1$ and $\angle G(j\omega) = -2$ rad at all ω

Lowpass to highpass, bandpass and bandstop

50. The normalized Butterworth prototype $G_{LP}(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$ has cutoff 1 rad/s. Replacing s by $50/s$ gives

- (a) $\frac{s^2}{s^2 + 1.414s + 2500}$
- (b) $\frac{s^2}{s^2 + 70.71s + 2500}$
- (c) $\frac{s^2}{s^2 + 70.71s + 50}$
- (d) $\frac{2500}{s^2 + 70.71s + 2500}$

51. In a two-way loudspeaker crossover, the tweeter (a $4\ \Omega$ resistor) is fed through a series capacitor C , giving the highpass $G_H(s) = \frac{4Cs}{4Cs + 1}$. Find C in μF so the -3 dB frequency is 3 kHz.

Answer: _____

52. A bandstop filter is built as the parallel sum of a first-order lowpass and a first-order highpass: $G(s) = \frac{10}{s + 10} + \frac{s}{s + 1000}$. Find $|G(j100)|$.

Answer: _____

53. A first-order lowpass $G_L(s) = \frac{\omega_c}{s + \omega_c}$ and a first-order highpass $G_H(s) = \frac{s}{s + \omega_c}$ with the same cutoff are connected in parallel and summed, $G = G_L + G_H$. What is G ?

- (a) A bandstop filter with a notch at ω_c
- (b) A bandpass filter centered at ω_c
- (c) $|G(j\omega_c)| = \sqrt{2}$, with unit gain elsewhere
- (d) $G(s) = 1$: every frequency passes unchanged

Butterworth filters and order selection

54. A second-order lowpass with no zeros has poles at $-10^4 \pm j10^4$ rad/s and unit DC gain. It is a Butterworth filter. Find its -3 dB cutoff ω_c in rad/s.

Answer: _____

55. A second-order Butterworth lowpass with $f_c = 100$ Hz has transfer function $\frac{\omega_c^2}{s^2 + a s + \omega_c^2}$. Find a in rad/s.

Answer: _____

56. A fifth-order Butterworth lowpass has $\omega_c = 1000$ rad/s. Find the real part of the pole pair closest to the imaginary axis.

Answer: _____

57. A third-order Butterworth highpass filter with cutoff frequency 100 Hz is used to suppress a 50 Hz hum. By how many dB is the hum attenuated? (Give a positive number.)

Answer: _____

58. Two identical second-order Butterworth lowpass sections, each with cutoff ω_c , are cascaded. Which statement about the cascade is correct?

- (a) The gain at ω_c is -6.02 dB, and the cascade is a fourth-order Butterworth filter with cutoff ω_c
- (b) The gain at ω_c is -3.01 dB, and the cascade is a fourth-order Butterworth filter with cutoff ω_c
- (c) The gain at ω_c is -12.04 dB, and the cascade is not a fourth-order Butterworth filter
- (d) The gain at ω_c is -6.02 dB, and the cascade is not a fourth-order Butterworth filter

59. A second-order Butterworth highpass has cutoff 6 kHz, so $|G(j\omega)|^2 = 1/(1 + (\omega_c/\omega)^4)$. At what frequency, in Hz, is its gain -30 dB?

Answer: _____

60. A fourth-order Butterworth lowpass must lose at most 1 dB at 1 kHz. What is the smallest cutoff frequency f_c (the -3 dB frequency), in kHz?

Answer: _____

61. A Butterworth lowpass filter with cutoff frequency 2 kHz must attenuate a 6 kHz signal by at least 35 dB. Enter the smallest order n that meets this.

Answer: _____

4. Modulation (week 11)

Amplitude modulation and demodulation

62. An antenna's length must be comparable to the wavelength $\lambda = c/f$ of the signal it transmits, with $c = 3 \times 10^8$ m/s. Find the wavelength of a 3 kHz telephone audio signal sent directly as an electromagnetic wave, in km.

Answer: _____

63. The message $u(t) = 2\cos(1000t) + \cos(2000t)$ is transmitted as $y(t) = u(t)\cos(10\,000t)$. Which list gives the positive frequencies (rad/s) in $y(t)$ with the amplitudes of the corresponding cosines?

- (a) 8000 (1), 9000 (2), 11 000 (2), 12 000 (1)
- (b) 1000 (2), 2000 (1), 10 000 (1)
- (c) 8000 (0.5), 9000 (1), 11 000 (1), 12 000 (0.5)
- (d) 10 000 (1), 11 000 (1), 12 000 (0.5)

64. A speech scrambler multiplies $u(t)$, bandlimited to 15 kHz, by $2 \cos(2\pi \cdot 15\,000 t)$ and passes the product through an ideal lowpass filter with cutoff 15 kHz and gain 1. A component $3 \cos(2\pi \cdot 4000 t)$ of $u(t)$ appears in the scrambled output as
- (a) $3 \cos(2\pi \cdot 19\,000 t)$
 - (b) $3 \cos(2\pi \cdot 4000 t)$
 - (c) $3 \cos(2\pi \cdot 11\,000 t)$
 - (d) $1.5 \cos(2\pi \cdot 11\,000 t)$

Frequency division multiplexing

65. A radio band runs from 3.000 MHz to 3.400 MHz. Each telephone message has bandwidth 3.4 kHz and is sent by DSB-SC, with no guard bands and no overlap allowed. What is the largest number of channels that fit in the band?

Answer: _____

66. Four messages, each bandlimited to 3 kHz, are sent by DSB-SC on carriers $f_1 < f_2 < f_3 < f_4$. The FDM spectrum must begin at 100 kHz, with 5 kHz guard bands between adjacent channels and no gaps otherwise. Find f_4 in kHz.

Answer: _____

67. Two messages $u_1(t)$ and $u_2(t)$ are each bandlimited to 5000 rad/s. The signal $b(t) = u_1(t) + u_2(t) \cdot 2 \cos(10\,000 t)$ is formed and then transmitted as $c(t) = b(t) \cdot 2 \cos(20\,000 t)$. Which band of positive frequencies does $c(t)$ occupy?

- (a) 5000 to 35 000 rad/s
- (b) 10 000 to 30 000 rad/s
- (c) 0 to 35 000 rad/s
- (d) 15 000 to 25 000 rad/s

Frequency and phase modulation, instantaneous frequency

68. The signal $y(t) = \cos(t^2)$ is a sinusoid with variable frequency, $y(t) = \cos(\omega(t)t)$ with $\omega(t) = t$. Find its instantaneous frequency at $t = 5$ s, in rad/s.

Answer: _____

69. A PM transmitter sends $y(t) = \cos(\omega_c t + k u(t))$ with $k = 0.5$ rad/V. Over $0 \leq t \leq 1$ ms the message is a ramp, $u(t) = 4000 t$ V. By how many Hz is the instantaneous frequency shifted from the carrier during the ramp?

Answer: _____

70. The message $u(t) = H(t)$ is sent once by FM, $y = \cos\left(\omega_c t + k \int_{-\infty}^t u\right)$, and once by PM, $y = \cos(\omega_c t + k u(t))$. Which description of the two signals for $t > 0$ is correct?

- (a) Both have frequency $\omega_c + k$ for $t > 0$.
- (b) Both multiply the carrier amplitude by $1 + k$ for $t > 0$.

- (c) FM: the frequency jumps to $\omega_c + k$ with continuous phase. PM: the phase jumps by k at $t = 0$ and the frequency is ω_c again for $t > 0$.
- (d) FM: the phase jumps by k at $t = 0$. PM: the frequency jumps to $\omega_c + k$.
71. A message $u(t)$ with spectrum zero for $|\omega| > W$ (and no DC component) is sent by FM, $y(t) = \cos\left(\omega_c t + k \int_{-\infty}^t u(\tau) d\tau\right)$, with $\omega_c \gg W$. For small but nonzero k , the spectrum of $y(t)$ consists of
- (a) lines at $\pm\omega_c$ plus sidebands in $\omega_c - W < |\omega| < \omega_c + W$, the same band as AM
 - (b) components spread over a band much wider than $2W$ around $\pm\omega_c$
 - (c) lines at $\pm\omega_c$ only
 - (d) the baseband spectrum $|\omega| < W$ plus lines at $\pm\omega_c$