# ELEC2302/9302 Signals and Systems — Quiz 1 formula sheet

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## Complex exponentials

$$e^{j\theta} = \cos\theta + j\sin\theta$$

A complex number in Cartesian and polar form:

$$z = x + jy = r\,e^{j\theta}, \qquad r = |z| = \sqrt{x^2+y^2}, \qquad \theta = \arg z.$$

## Inner product, energy and power

$$\langle u, y\rangle := \int_{-\infty}^{\infty} u(t)\,\overline{y}(t)\,\mathrm{d}t$$

$$E(u) = \int_{-\infty}^{\infty} |u(t)|^2\,\mathrm{d}t, \qquad P(u) = \lim_{W\to\infty}\frac{1}{2W}\int_{-W}^{W}|u(t)|^2\,\mathrm{d}t, \qquad u_{\mathrm{RMS}} = \sqrt{P(u)}.$$

If $u$ is periodic with period $T$, then

$$\langle u, y\rangle_P = \frac{1}{T}\int_0^T u(t)\,\overline{y}(t)\,\mathrm{d}t, \qquad P(u) = \langle u, u\rangle_P = \frac{1}{T}\int_0^T |u(t)|^2\,\mathrm{d}t.$$

## Standard signals

**Unit step (Heaviside).**

$$H(t) = \int_{-\infty}^{t}\delta(\varsigma)\,\mathrm{d}\varsigma = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

**Pulse of width $T$.**

$$p_T(t) = \begin{cases} 1, & |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

$$p_T(t) = H\left(t+\tfrac{T}{2}\right) - H\left(t-\tfrac{T}{2}\right), \qquad \frac{\mathrm{d}}{\mathrm{d}t}H(t) = \delta(t).$$

## The Dirac delta

**Sifting property.**

$$\int_{-\infty}^{\infty}\delta(t-\tau)\,y(t)\,\mathrm{d}t = y(\tau).$$

**Scaling.**

$$\delta(\alpha t) = \frac{1}{|\alpha|}\,\delta(t).$$

## Convolution

$$(h * u)(t) := \int_{-\infty}^{\infty} h(t-\tau)\,u(\tau)\,\mathrm{d}\tau = (u * h)(t)$$

$$(u*\delta)(t) = u(t), \qquad \big(u*\delta(t-T)\big) = u(t-T), \qquad h_1 * (h_2 * h_3) = (h_1 * h_2) * h_3.$$

## Impedances

$$Z_R = R, \qquad Z_L = j\omega L, \qquad Z_C = \frac{1}{j\omega C}.$$